

## Accelerated Algebra II: Trigonometry ~ Part II

### 12.4 – Extending Trigonometry

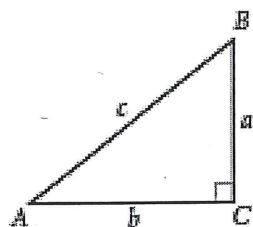
Objectives:

1. Define Angle of Rotation.
2. Extend definition of trigonometric functions to angles of any measure.
3. Draw a reference triangle.

Review

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**Example 1:** Find the sine, cosine, and tangent of angle A below:

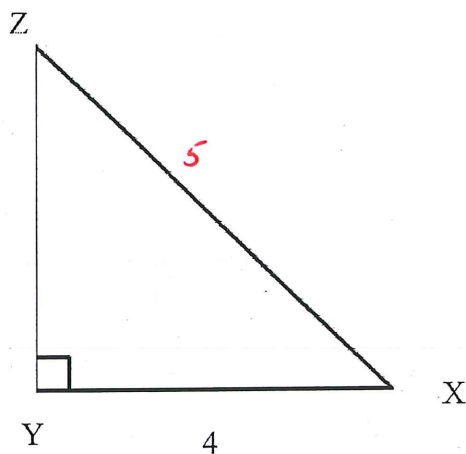


$$\sin A = \frac{a}{c}$$

$$\cos A = \frac{b}{c}$$

$$\tan A = \frac{a}{b}$$

**Example 2:** Find the missing side of the right triangle below, then find the values below as fractions and decimals:



$$\sin Z = \frac{4}{5} = .8$$

$$\sin X = \frac{3}{5} = .6$$

$$\cos Z = \frac{3}{5} = .6$$

$$\cos X = \frac{4}{5} = .8$$

$$\tan Z = \frac{4}{3} = 1.\bar{3}$$

$$\tan X = \frac{3}{4} = .75$$

What relationship do you notice between the sine and cosine of an angle and its tangent?

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

## Accelerated Algebra II: Trigonometry ~ Part II

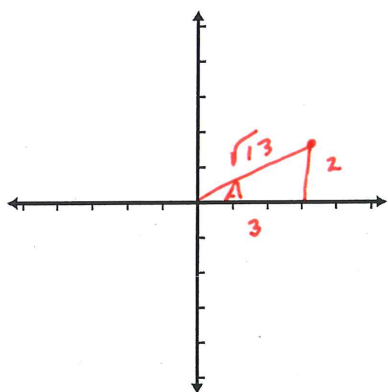
**Standard Position:** An angle positioned with one side on the positive  $x$ -axis.

**Terminal Side:** The side of an angle in standard position that is not on the positive  $x$ -axis.

**Reference Angle:** The acute angle between the terminal side of an angle in standard position and the  $x$ -axis.

**Reference Triangle:** A right triangle that is drawn connecting the terminal side on an angle in standard position to the  $x$ -axis.

**Example 3:** Plot the point (3,2) on the graph below:



- Draw the reference triangle.
- Find the lengths of the sides of the reference triangle as fractions and decimals:

$$\begin{aligned}2^2 + 3^2 &= c^2 \\4 + 9 &= c^2 \\13 &= c^2 \\\sqrt{13} &= c\end{aligned}$$

c) Find the sine, cosine, and tangent of the reference angle.

$$\sin A = \frac{2}{\sqrt{13}}$$

$$\cos A = \frac{3}{\sqrt{13}}$$

$$\tan A = \frac{2}{3}$$

d) Find the measure of the reference angle.

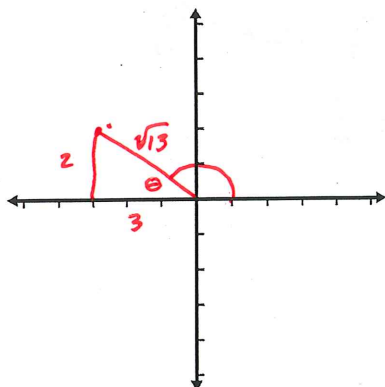
$$\begin{aligned}A &= \sin^{-1}\left(\frac{2}{\sqrt{13}}\right) \\A &= 33.69^\circ\end{aligned}$$

$$\begin{aligned}A &= \cos^{-1}\left(\frac{3}{\sqrt{13}}\right) \\A &= 33.69^\circ\end{aligned}$$

$$\begin{aligned}A &= \tan^{-1}\left(\frac{2}{3}\right) \\A &= 33.69^\circ\end{aligned}$$

## Accelerated Algebra II: Trigonometry ~ Part II

**Example 4:** Plot the point  $(-3, 2)$  on the graph below.



- Draw in the angle that measures from the positive  $x$ -axis to the ray from the origin through  $(-3, 2)$ .
- Draw in the reference triangle.
- Find the sides of the reference triangle.

d. Find the sine, cosine, and tangent of the reference triangle:

$$\sin A = \frac{2}{\sqrt{13}}$$

$$\cos A = \frac{3}{\sqrt{13}}$$

$$\tan A = \frac{2}{3}$$

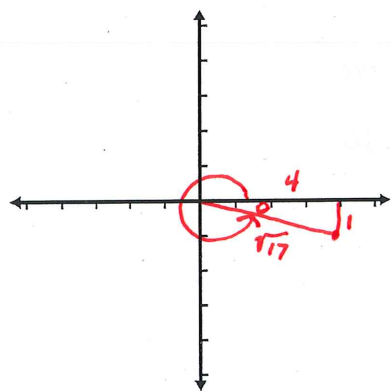
e. Find the measure of the reference angle.

$$\sin^{-1}\left(\frac{2}{\sqrt{13}}\right) = 33.69^\circ$$

f. Use the measure of the reference angle to find the measure of the angle you drew in part a.

$$180 - 33.69^\circ = 146.31^\circ$$

**Example 5:** Find the sine, cosine, and tangent of an angle, measured counterclockwise, from the positive  $x$ -axis from the origin through  $(4, -1)$ .



$$\begin{aligned} 1^2 + 4^2 &= c^2 \\ 1 + 16 &= c^2 \\ 17 &= c^2 \\ \sqrt{17} &= c \end{aligned}$$

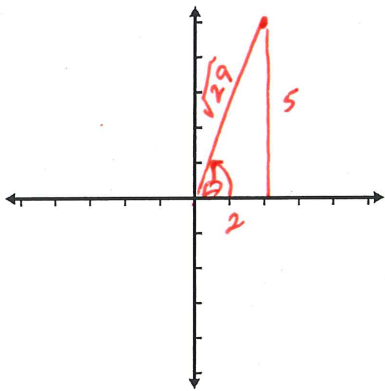
$$\sin \theta = \frac{1}{\sqrt{17}} \quad \cos \theta = \frac{4}{\sqrt{17}} \quad \tan \theta = \frac{1}{4}$$

$$\theta = \tan^{-1}\left(\frac{1}{4}\right) = 14.036^\circ$$

$$\begin{aligned} \text{Angle} &= 180 - 14.036^\circ \\ &= 165.964^\circ \end{aligned}$$

## Accelerated Algebra II: Trigonometry ~ Part II

**Example 6:** Find the sine, cosine, and tangent of an angle, measured counterclockwise, from the positive x-axis from the origin through (2, 5).

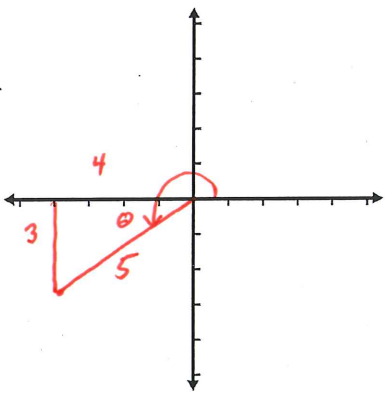


$$\begin{aligned} 2^2 + 5^2 &= c^2 & \sin \theta &= \frac{5}{\sqrt{29}} & \cos \theta &= \frac{2}{\sqrt{29}} \\ 4 + 25 &= c^2 & \tan \theta &= \frac{5}{2} \\ 29 &= c^2 \\ \sqrt{29} &= c \end{aligned}$$

$$\theta = \tan^{-1}\left(\frac{5}{2}\right)$$

$$\theta = 68.199^\circ$$

**Example 6:** What measure describes an angle, measured counterclockwise, from the positive x-axis to the ray from the origin through (-4, -3)?



$$\begin{aligned} 3^2 + 4^2 &= c^2 \\ 9 + 16 &= c^2 \\ 25 &= c^2 \\ 5 &= c \end{aligned}$$

$$\tan \theta = \frac{3}{4}$$

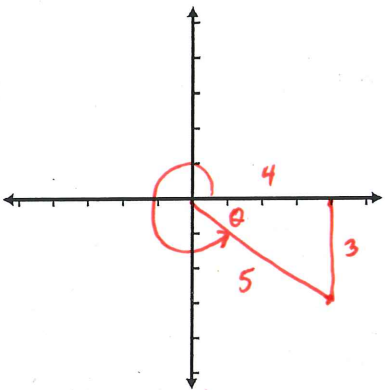
$$\theta = \tan^{-1}\left(\frac{3}{4}\right)$$

$$\theta = 36.870^\circ$$

$$A = 180 + 36.870^\circ$$

$$\boxed{A = 216.870^\circ}$$

**Example 7:** What measure describes an angle, measured counterclockwise, from the positive x-axis to the ray from the origin through (4, -3)?



$$\tan \theta = \frac{3}{4}$$

$$\theta = 36.870^\circ$$

$$A = 360 - 36.870^\circ$$

$$A = 323.130^\circ$$